Needed adult numeracy and critical statistical skills: A view from international skill frameworks, and implications for education

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Experience
2011 - present, Chair, Department of Human Services, University of Haifa
2011 - 2013, President-Elect, International Association for Statistical Education (IASE)
2008 - 2013, Chair, Numeracy Expert Group, OECD's Programme for International Assessment of Adult Competencies (PIAAC)
1998 - 2005, Member, Numeracy Group, Adult Literacy and Lifeskills Survey (ALLs)
1995 - present, Advisory Board Member, Int'l Statistical Literacy Project (ISLP)

Selected Publications


Introduction
This talk examined in broad strokes several issues related to the development of skills which are part of the human capital of every country, with a focus on adult numeracy and mathematical literacy, and the subtopic of statistical literacy which is a part of numeracy but has some unique components.

The motivation for this paper (and for the talk on which it is based) stems from the recent launch of the OECD's new study of adult skills, the Programme for International Assessment of Adult Competencies (PIAAC), which is focused on the competencies of adults aged 16-65 and is now underway in 35 countries on a cyclic basis (somewhat similar to PISA). Many countries have joined the PIAAC program, including Japan, but a great number of professionals in these countries do not have a sufficient background in such assessments.

With the above in mind, this paper (Slide 1) aims to familiarize professionals and stakeholders in the logic underlying adult skills assessments in general, and assessment of numeracy in particular. The paper later examines some of the educational ramifications of a model of adult numeracy and statistical literacy, which are targeted by recent international frameworks of adult competencies.

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1. Introduction - The challenge: Human capital (From "education" to "skills/competencies")
2. Analytic potential of skills surveys
3. About Numeracy, mathematical and statistical literacy, and "critical" skills
4. Issues & Implications: Instruction & Assessment
5. Discussion, Q & A

Countries interested in knowing what skills (e.g., literacy) their citizens have, i.e., the human capital available to them, have had to rely for years on crude proxy measures, such as the number of years of education completed. However, over the last 30 years, a need has emerged to have better knowledge about the actual competencies of adults, not just about their ability to read or write, simply viewed. Generating such knowledge requires the use of direct cognitive measures of skills, i.e., tests. Particular attention has been given to adults' ability to perform various real-world tasks in three related but separate life roles, i.e., being producers of information (e.g., writing a letter, verbal expression), consumers of information (e.g., reading a newspaper), and decision-makers (e.g., medical choices).
The actual range of issues covered in the international assessments of adult skills is very broad, and includes three clusters (see Slide 3). The first is information about distributions of cognitive skills in the population and in subgroups of interest (e.g., which may be the target of intervention, should their skills be relatively low), and trends in this regard. The second is antecedents and correlates that may help to explain the skill distribution or how it is associated with various background factors. The last and a very important cluster relates to social outcomes and variables of national importance, such as whether people are employed or unemployed, how long they take to find a new job, and how these and other outcomes are associated with actual competencies and the other correlates.

Surveys of adult skills have a distinguished history starting about 30 years ago (Slide 4), using a methodology (Slide 5) that involves household surveys where a nationally representative sample is chosen with people visited at their homes and asked to respond to tasks using authentic real-world stimuli and questions reflecting issues that an adult might actually have to cope with in his or her life.

The conceptual frameworks underlying early surveys have pertained to three components of literacy (Prose, Document, Quantitative), based on the Kirsch-Mosenthal model. However, starting with the Adult Literacy and Lifeskills survey (ALLs) in 2003-2006 (Murray, Clermont, & Binkley (2005), a range of additional constructs has been introduced into the thinking of policy-makers and assessment specialists (Slide 6).
Of most interest for the present talk is the introduction of a broad construct called "numeracy" instead of the important but more restricted notion of "quantitative literacy" that was measured in prior studies such as the Int'l Adult Literacy Survey (IALS).

In terms of methodology, PIAAC employs both a computer-based and paper-based adaptive testing scheme, in order to collect information in an efficient way yet be suitable to adults with limited or no experience in using computers.

Analytic potential of adult skills surveys

Studies of adult skills such as ALLs and PIAAC have a vast analytic potential, much greater than studies of students, given the different range of correlates examined and the ability to examine the link between measured cognitive skills and social outcomes of national importance (i.e., the right-hand panel in Slide 3). Such studies can provide diverse types of data and answer questions that cannot be answered in any other way. Slides 8 and 9 provide two simple illustrations (both are based on ALLs data, as official PIAAC findings have not been released yet).
distribution in document literacy in seven of the countries that participated in the first wave of ALLs.

Analytic potential 1: Skills distributions, and education

One can see in Slide 8 marked differences not only in average skill levels but also in the internal distribution. Especially noticeable is the ability of such a survey to identify populations at levels 1 and 2, which are usually considered "at risk" and associated with elevated levels of negative social outcomes (e.g., unemployment).

Analytic potential 2: Mean numeracy scores on a scale with range 0 to 480 points, by level of educational attainment, populations aged 16 to 65, 2003

The above are but two examples. Overall, PIAAC and the prior studies on which it is based have the potential to answer many questions of interest both to policy makers and to professionals and scholars interested in people and their skills (e.g., in diverse areas such as labor and training, education, industry and commerce, health and human services, rehabilitation and prisons). Slide 10 presents a broader list of key policy-related questions or topics.

Adult numeracy

We now take a closer look at numeracy, one of the key competencies targeted both by PIAAC and prior surveys of adult skills. Numeracy (see Slide 11) has been for many decades a key skill of critical importance for adults, and this is increasingly so in the information age. Given its importance, it
has also been addressed, though with a slightly different framework, under the notion of Mathematical Literacy in the OECD's PISA assessment program which focuses on the skills of 15-year old students.

Numeracy has been defined in PIAAC as follows: "The ability to access, use, interpret, and communicate mathematical information and ideas, to engage in and manage the mathematical demands of a range of situations in adult life." This definition is consistent with the conceptualizations both of the broad notion of "literacy in the information age" and of "competency" as described earlier.

Let me note that the list shown in Slide 12 was originally designed to serve as a map for the production of assessment items, i.e., in an ideal world, a full test of numeracy should include items that cover all combinations of the different facets in the list. However in addition, it sketches in broad strokes educational goals, i.e., teachers and education administrators should ask themselves to what extent students are ready to cope with tasks covering all the facets in the list.

![Numeracy in PIAAC: Operationalization](slide_11)

Going beyond the view of "numeracy" presented above, to be able to conceptualize the building blocks of numeracy, PIAAC Numeracy Expert Group has assumed that in the real world, adults are required to be able to act in a numerate way or demonstrate numerate behavior (Slide 11) in diverse contexts and situations, i.e., home, commerce, civic or community action, workplace and further learning. Slide 12 presents a detailed operationalization of numeracy, achieved through this sub-construct of "numerate behavior."

Before continuing, let me elaborate on some of the elements in the definition of numeracy listed in Slide 11. Due to space limits I highlight just three of the terms in the description of adult numeracy:

- "Interpret" reminds us of the role of adults as critical consumers of quantitative information, as interpreters of a wide range of quantitative messages. Such information is often presented via different types of texts. Yet, texts are often seen by mathematics teachers as a distraction, as external to the world of pure mathematics, and shunned from the classroom.

- "Engage" is not about having engaging instruction, as some may think, but about preparing learners to
effectively engage a very wide range of real-life situations that present mathematical demands. To engage such tasks successfully, one needs not only a range of cognitive skills and knowledge bases, but also positive or supporting dispositions, i.e., beliefs, attitudes, and a critical stance, coupled with productive habits of the mind. We want people to feel comfortable about being able to approach and cope with a range of tasks, including tasks that involve ambiguity, call for decision making and solving problems embedded in real contexts, and the like.

- "Manage" refers to the fact that adults do not normally “solve” problems as in a math class. Most numeracy situations do not have "solutions" that can be classified as right or wrong. Rather, adults manage situations, and can decide on one of several courses of action, based on their assessment of personal goals and situational demands, severity of the consequences, and personal and situational resources.

The conceptual model of adult numeracy assumes that the extent to which people can act in a numerate way or demonstrate numerate behavior depends on several underlying enabling factors and processes (see Slide 13). Detailed descriptions of these enabling processes can be found in Kilpatrick (2001) and in PIAAC Numeracy Expert Group (2009). Some are discussed in more detail below in connection with a specific model of numeracy and statistical literacy.

### Enabling factors and processes
(what underlies numerate behavior)

1. Mathematical/statistical knowledge and conceptual understanding
2. Adaptive reasoning and [mathematical] problem-solving skills
3. Literacy skills
4. Context/world knowledge
5. Beliefs & attitudes
6. Numeracy-related practices & experience

#### Numeracy and statistical literacy: Further examples and a model

In this section I want to further elaborate about some of the enabling processes described in Slide 13, and go beyond adult numeracy to also address statistical literacy, a construct that on the one hand is part of adult numeracy, but on the other hand extends beyond it, given known differences between mathematics and statistics (Gal, 2002). These examples and the discussion that follows can help to broaden the view of the skills needed of adults which are part of human capital in the information age.

Slide 14 shows a portion of an article taken from the largest circulated newspaper in my country, Israel (written in Hebrew, which is read from right to left). It discusses poverty and income gaps. This example illustrates a typical newspaper article reporting quantitative information about a socially meaningful topic, and shows that such articles contain multiple components. Key information is reported via textual means (e.g., the top headline argues: "Every fifth worker is below the poverty line"), and it includes a graph and a table with some figures. In addition, on the left side is a separate critical
commentary on the main article by a separate commentator.

Slide 14

Slide 15 shows two excerpts from the English-based version of the *Asahi Shimbun*, a highly respected Japanese newspaper, which I scanned when preparing this talk. Surprisingly, two separate news stories appeared one above the other, both reporting a percentage figure. Can you tell which is based on real data, and which is using percentages as a figure of speech (i.e., not based on actual quantitative information)?

These are just two of many possible examples, i.e. they do not illustrate, for example, issues regarding the tasks and cognitive processes facing adults when they have to interpret medical or financial information and understand future projections or risk levels. Still, they are sufficient to make three related but separate important points:

1. A key aspect of mathematical literacy, numeracy, or statistical literacy is literacy, i.e., language skills. Without being able to comprehend and interpret text-based arguments, one cannot be considered mathematically literate.

2. Readers need to be able to access contextual knowledge in order to put in perspective and fully understand the quantitative information or arguments they encounter.

3. Readers need to understand that quantitative arguments are made by diverse types of sources, and be reflective and critical in order to interpret things correctly.

Surprisingly, despite the points made above regarding the central role of literacy and critical interpretation for effective management of numeracy or statistical literacy tasks, it seems that mathematics and statistics curricula seldom pay much attention to text reading or to critical interpretation of texts as an essential aspect of teaching mathematics or statistics. Further, my interactions with mathematics and statistics teachers at the high-school and college levels show that they invariably see text as a nuisance, as a distraction that takes away from the precision of mathematical and statistical symbolic representations in formulas and proofs. Yet, text understanding is a primary gateway and an inherent component of numeracy, mathematical literacy and statistical literacy.

In order to further our understanding of the
challenges involved in developing numeracy and statistical literacy competencies, let me go a step beyond the assessment framework described earlier (i.e., the numeracy framework developed for PIAAC) and present a broader model (see Slide 16) which is geared for teaching and instructional planning purposes. I originally developed this to describe the building blocks of statistical literacy but it is extended here to describe numeracy and mathematical literacy more generally.

Integrated Model:
Numeracy & Statistical literacy

- Cognitive bases
  - Literacy
  - Statistics / probability
  - Mathematics
  - Critical questions
  - World/context knowledge
  - Technology

- Dispositional bases
  - Attitudes
  - Beliefs
  - Critical stance

- Enabling processes
  - Habits of mind
  - Adaptive reasoning...

Integrated competency

Slide 16

The model in Slide 16 promotes the idea that in order to develop an integrated competency (i.e., "numeracy" or "mathematical literacy" or "statistical literacy"), people need to possess both multiple (cognitive) knowledge bases and supporting dispositional bases (attitudes, beliefs, a critical stance, and other factors). These in turn are supported by a range of enabling processes and factors. (Details regarding the original conceptualization for the area of statistical literacy can be found in Gal (2002), with extensions for the area of "probability literacy" in Gal 2004, and additional discussions pertaining to numeracy in Gal (1998) and in PIAAC Numeracy Expert Group (2009).

Given space limitations, let me just highlight two of the aspects in this model that were not evident in the PIAAC model of numeracy: the need for possession of "critical questions" (a cognitive knowledge base) and a supporting "critical stance" (a dispositional base). Slide 17 lists some typical "worry questions" that a person needs to know about (a knowledge base) and be willing to activate (a motivational dispositional base).

Sample "worry/critical questions" about statistical/quantitative messages

1. Where did the data (on which this statement is based) come from? What kind of study was it?
2. Was a sample used? How was it sampled? How many people did participate? Is the sample large enough? Is the sample biased? How?
3. How reliable or accurate were the instruments or measures (tests, questionnaires, interviews) used to generate the reported data?
4. Are the reported statistics appropriate for this kind of data, e.g., was an average used to summarize ordinal data? Could outliers cause a summary statistic to misrepresent the true picture?
5. Is a given graph drawn appropriately, or does it distort trends?

Slide 17

Slide 18 presents one more example that helps to illustrate the need for critical interpretation of quantitative messages, as well the complexity of doing so. This example originated in the results from the international TIMMS survey of 1998. As part of the TIMSS report, Mullis, Martin, Beaton, Gonzalez, Kelly, & Smith (1998) reported performance levels of students in their final year of schooling (usually grade 12) on a task directly related to numeracy and statistical literacy: Learners were asked to explain whether a reporter’s statement about a “huge increase” was a reasonable interpretation of a bar graph showing the number of robberies in two years that was manipulated to create a specific impression. The graph included a bar for each year but with a truncated scale, causing a small difference between the years to appear to be large.
A TV reporter showed this graph and said:
"There has been a huge increase in the number of robberies this year"
Do you consider the reporter's statement to be a reasonable interpretation of the graph? Briefly explain.

Performance levels varied across countries; on average, less than half of all graduating students appeared to be able to cope (at least partially) with this task that exemplifies one of the most basic skills educators usually use as an example for a statistical literacy skill expected of all citizens: i.e., ability to detect a discrepancy between displayed data and a given interpretation of these data. Keeping in mind that in many countries a sizable proportion of students drop out or leave before the final year of high school, the overall percentage of all school leavers who can cope with such tasks is bound to be even lower.

This task was later used in PISA 2003 (there called "Robberies") and subsequently adapted for use in the ALLs survey of adult skills. In all countries, the average performance was at the 30% to 60% level, suggesting that many students as well as adults either have difficulty or are unable to cope well with reading of a rather simple graph, when they are asked to do more than literal reading of information in the graph. I argue that the data pertaining to the performance on the Robberies task shown in Slide 18 demonstrate that numerate behavior which combines in an integrated way both cognitive and dispositional bases shown in the model in Slide 16 cannot be expected to naturally evolve out of learning regular school mathematics.

Summary and implications

A discussion about the meaning of numeracy and mathematical literacy and its place in mathematics education is a timely one. The current attention to this topic is fueled in part by a force that many mathematics educators and mathematicians may see as external to mathematics education itself, i.e., the emphasis on mathematical literacy and on numeracy in the OECD's two key assessment programs, PISA (students) and PIAAC (adults).

Summary
1. New emphasis on outputs of educational system (i.e., in terms of competencies reflecting external real-world demands) as opposed to inputs.
2. Need to understand the broad nature of competencies expected of adults (citizens, workers) and the connections between literacy, numeracy, & ICT.
3. The potential of large-scale assessments such as PIAAC and PISA to describe the distribution of skills, correlates, and contribution to social and economic outcomes.
4. The complex and varied nature of what is required from an adult to be a "critical consumer" of statistical and quantitative information.
important to teach?" I believe a broader question should be asked: What are the skills or competencies needed by and expected of citizens from all walks of life for effective functioning in the information age [at work, at home, in civic life, etc]?

Implications
Set educational policy & instructional goals based on a unified view/model of the competencies that adults need to function effectively in an information-rich, dynamic world.

"Unified" in terms of:
- Instruction that connects mathematics, statistics, probability, literacy, and ICT technology in instruction
- Develops both cognitive and non-cognitive elements
- Link of instructional tasks to real world demands (more authentic instruction, prep for skill transfer)
- Assessment: that can serve program evaluation purposes
  - change in cognitive skills & skill transfer
  - attitudes & beliefs, empowerment

Evaluators are now called upon by external stakeholders to demonstrate that virtually all of their students are coming out with a broad range of needed competencies. So far, cumulative results both from PISA and from large-scale surveys of adult skills (PIAAC Numeracy Expert Group, 2009) suggest that too many people, and students, are not engaging real-life mathematical or statistical tasks very well. Thus, whatever is being done in schools is not working as well as it should. Why? Is it because we are not doing well enough in teaching students to think like mathematicians (or statisticians) and should try harder in this regard?

I believe that we should look for different answers. Given space limits, let me emphasize just two: we need to better understand the nature of the target competencies themselves, and be aware of the cumulative research findings regarding the complexity of skill transfer and of the many factors affecting people's ability to cope with new kinds of mathematical or functional tasks in different life contexts (Lovett, & Greenhouse, 2000; Burke, & Hutchins, 2007).
term 'numeracy' and not 'mathematical literacy', in order to posit numeracy and literacy as two related but separate modes of thinking about, knowing, and interacting with the world).

To develop a transferable competency such as numeracy or mathematical literacy, and to increase the chance our graduates can autonomously engage a wide range of real-life mathematical or statistical tasks and situations, we need to rethink the mix of tasks used in instruction, and the associated teaching sequences and assessment methods. Among other directions, we need to increase the amount of tasks that:

- involve ill-structured problems similar to real-life tasks,
- contain text-based messages conveying various quantitative and statistical arguments or requiring critical interpretation of texts,
- present statistical information of the kinds normally encountered in the media or in workplace and civic action contexts,
- demand the kinds of coping behaviors that adults are called upon to demonstrate in real life, including the use of technology for accessing, sifting through and organizing quantitative information.

References


